Dimensional

RESEARCH

Empirical Evidence in the Fixed Income Markets

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Research

There are many aspects to consider when investing in fixed income, including the term and credit dimensions of expected bond returns, inflation risk, currency exposure, and bond trading. This survey is organized around these topics, with discussion of the empirical findings related to each, and their practical implications for fixed income investment.

In the following discussion, we will use the terms such as yield, forward rate, and duration. A brief review of these concepts is provided in the appendix for readers who are interested in more details.

I. TERM

The term structure of interest rates—often presented as a *yield curve* that plots the yields of bonds across maturities—plays a central role in the fixed income markets. From current yields, one can compute a sequence of forward rates at different maturities. The calculation essentially compares two hypothetical investments: One is to purchase a bond with longer maturity; the other option is to buy a shorter-term bond and, once it matures, roll it over to a new bond with a maturity that covers exactly the remaining life of the longer-term bond. The forward rate, then, is a "future" interest rate on that new bond that will set the two investment returns the same. The forward looking nature of these calculations naturally leads to the question of what information forward rates contain about future interest rates and bond returns. Algebraically, Fama (1976) shows that the forward rate in excess of the current short rate can be written as a combination of the expected change in future spot rate and the expected term premium on bonds of matched maturity. There are different views, however, on the magnitude and relative importance of each component.

FORWARD RATE EXAMPLE

Suppose the current yield of a zero-coupon bond is 2% for one-year maturity and 3% for two-year maturity, both continuously compounded. Then the one-year forward rate one year from now is (3% × 2) – 2% = 4%.

To interpret this forward rate, consider two hypothetical investment options: one is to purchase a two-year bond; the other is to buy a one-year bond and roll it over to another one-year bond once it matures. Then the 4% forward rate is a "future" interest rate on the second one-year bond that will set the two investment returns the same—earning the current one-year rate in the first year and the forward rate in the second year yields (2% + 4%)/2 = 3% per annum, which is equal to the current two-year rate.

(For illustrative purpose only.)

The pure expectation hypothesis (PEH) states that forward rates are the market expectations of future spot rates. If this holds, the only reason for an upward sloping yield curve is an expectation of higher spot rates in the future. The PEH implies the yield on a long-term bond equals the average of expected future short-term rates over its life. Thus, if the PEH were true, expected term premiums would be zero—the expected return from holding a long-term bond to maturity would be the same as a strategy that holds a shorter-term bond to maturity and, at maturity, rolls to another shorter-term bond with a maturity that covers the remaining life of the long-term bond. This is because, under the PEH, future rates are expected to move in a way that offsets any initial yield differences.

Term premium is the difference in expected return between long-term and short-term bonds.

The PEH can be amended to allow for the existence of a constant term premium resulting in what is called the expectation hypothesis (EH). That is, the EH expresses forward rates as a combination of expected spot rates and time-invariant expected term premiums, which is less restrictive than the PEH. Nevertheless, both expectation hypotheses imply that variation in forward rates is entirely due to revisions in expectations about future interest rates.

PEH IN THREE EQUIVALENT FORMS

1. Forward rate = expected future spot rate.
   In the above example, it means that the one-year rate is expected to be 4% one year from now.

2. Long-term yield = average (expected future short-term yields).
   With the same example, the 3% on the two-year bond is the average of the one-year yields over the next two years—2% for the first year and 4% expected for the second year.

3. Expected term premium = 0.
   To see this is the case, we can compute the expected one-year holding period return on the two-year bond in the example. After being held for a year, it becomes a bond with one year left to maturity and an expected yield of 4%. So the expected holding period return for one year, (3% × 2) – 4% = 2%, equals the return on the one-year bond over the same period, i.e., no term premium.

The EH can be similarly expressed in these three equivalent forms but allows a constant term premium in the equations.

In contrast, an alternative view is that forward rates contain information about expected term premiums and not just information about expected future spot rates. Under this view, at a single point in time, differences in forward rates across maturities have information about differences in expected returns between different duration bonds. Further, under this view, term premiums can vary through time and changes in forward rates through time should contain information about the time varying nature of expected term premiums. Clearly, this view contradicts both the PEH and the EH, as it allows for term premiums to be positive and vary through time.

Whether the PEH, EH, or existence of a reliable relation between forward rates and term premiums better explains historical interest rate data is an empirical question. This question has been analyzed since the 1970s and the short answer is empirical data strongly rejects PEH, does not support EH, and supports the existence of a reliable relation between forward rates and expected term premiums. The research implies that the shape of the yield curve has information about differences in expected returns between different duration bonds.

Early studies documented the existence of positive term premiums (e.g., McCulloch, 1975; Fama, 1984a).[1] Their results reject the PEH (which predicts no term premiums) but do not explicitly address whether the EH holds. To test the EH, subsequent researchers focused on the informational content in yield curves. If a steep yield curve (high forward rates) implies higher expected excess returns on longer maturity bonds, the EH is rejected. Why? It indicates high forward rates are predicting high term premiums rather than higher future spot rates. Empirically, the relation between forward rates and expected term premiums has been reliable, while the evidence on the predictability of future interest rates has been mixed and weak. As we will discuss later, this finding has important investment implications beyond the testing of expectation hypotheses.

Fama (1984b) used data from 1959 to 1983 to analyze the informational content in yield curves. He found reliable evidence that forward rates contain lots of information about expected term premiums. When yield curves have been steep, there has been a reliable difference in subsequent average returns between longer and shorter maturities; whereas when the yield curve is flat, subsequent differences in returns have been smaller. He also found that forward rates show some explanatory power as predictors of one-month spot rates for one month in the future. Taken together, forward rates contain lots of information about term premiums and much less information about future spot rates. This is an empirical rejection of the EH.

What about the evidence of predictability over the very short term? Fama (1976) found that even after controlling for the variation in term premiums, forward rates still do not outperform the simple method of using the current interest rate as a predictor of future spot rates. This may be because spot interest rates tend to be highly persistent at short lags, and forward rates do not add additional explanatory power over lagged spot rates.

Since Fama (1984b), the positive relation between forward rates and expected term premiums has been documented in many studies. In particular, while the results in Fama (1984b) applied to the short end of the term spectrum, they were extended to longer maturity bonds by Fama and Bliss (1987) and Campbell and Shiller (1991), and have held up to out-of-sample tests on different data sets (e.g., Davis, 2000; Fama, 2003). In a related work, Cochrane and Piazzesi (2005) proposed a single-factor model based on a linear combination of forward rates to explain the variation in excess returns across different maturities.

Along with the empirical findings, many theoretical models for the evolution of interest rates have been proposed. The goal for term structure modeling is to be flexible enough to reproduce the empirical findings, yet tractable enough to provide insights into yield curve behavior. From a practical perspective, term structure models have proven useful for purposes such as interest rate derivatives pricing and risk management. They generally have not provided reliable forecasting power for future interest rates (Duffee, 2002).

TERM STRUCTURE MODELS

Short rate models specify the dynamics of one point (short rate) on the yield curve, and derive other points from the short rate.

Examples in this class include **one-factor models** like Vasicek (1977) and Cox, Ingersoll, and Ross (1985). These models are formulated to capture some characteristics of interest rate movements, but they have proven to be too simple to generate many of the features observed in historical bond data, such as time-varying term premiums (Chan et al., 1992).

Duffie and Kan (1996) generalized earlier models to a multifactor framework where the short rate depends on multiple variables. While the so-called **affine models** show more flexibility to fit real data, they still have limitations due to the specific structure imposed for tractability (Dai and Singleton, 2000).

1. For a comprehensive review of the early tests on expectation hypotheses, see Melino (1988).

Term spread is the difference between long-term and short-term interest rates. The more steeply the yield curve slopes upward, the wider the term spread.

Is the information in current yield curves about future return differences useful to potentially increase expected returns? The answer is yes. Instead of seeking to maintain a constant duration, a flexible investor can dynamically vary their duration within a window: shortening maturities when term spreads are narrow and term premiums are expected to be small; extending them when term spreads are wide and indicate a larger term premium. Data (both live and simulated) shows that such a variable maturity approach has led to a better tradeoff between return and expected volatility than strategies that maintain a constant term exposure. Further, a global allocation in fixed income enhances diversification and also broadens the opportunity set for varying maturities across and within yield curves. A dynamic approach that considers the tradeoffs between increased diversification and increased expected returns can be a reliable and robust way to add value in a global bond strategy. Plecha and Rodriguez (2011) and Kolerich and Rodriguez (2011) provided a summary of how this information can be applied in real-world portfolios.

II. CREDIT

Besides maturity, credit quality is another driver of expected return in fixed income securities. The yields of two bonds that are identical in all respects except for credit quality can be different—this difference is referred to as a credit spread. As observed in the markets, credit spreads have not been constant over time. An interesting question, then, is what causes credit spreads to vary and whether its variation tells us something about future expected returns.

Credit spread is the difference in yield between bonds of similar maturity but different credit quality.

Credit premium is the difference in expected return between bonds of similar maturity but different credit quality.

Theoretically, credit spreads should be determined by the probability of default, expected loss in the event of a default, expected premium as a compensation for the potential loss from defaults, and possibly other factors such as liquidity and taxes.[2] Because the components of credit spreads are not directly observable, empirical studies have attempted different methods to disentangle their effects. The results suggest that the default-related components—i.e., the probability of default, the expected loss in the event of a default, and the premium as compensation for bearing that risk—play a large role in the decomposition of credit spreads.

Longstaff, Mithal, and Neis (2005) used credit default swaps data to directly measure the default-related component of credit spreads. Based on their estimation, the default component makes up a majority of credit spreads across all credit ratings—the ratio of the default component to the total spreads increases from 62% to 89% as credit quality declines from AAA/AA-rated to BB-rated. Covitz and Downing (2007) regressed corporate spreads in commercial paper markets on various proxies for liquidity and credit quality. They reached a similar conclusion that credit quality appears to be the more important determinant of spreads. Elton et al. (2001) addressed tax effects explicitly and found that a substantial portion of corporate spreads remains after accounting for tax premiums and expected default loss; they suggest that the remaining part can be largely attributed to compensation for bearing systematic risk.

What does this mean for credit premiums? Studying an extensive data set covering the 1866–2008 period, Giesecke et al. (2011) provided a long-term historical perspective on US corporate bond returns. On average, credit premiums make up half of the credit spreads—resulting in an estimated premium of approximately 1% historically.

Variation in credit spreads can result from changes of any one or a combination of their components. For example, wide spreads could mean an increased default probability, a larger premium demanded by investors (possibly due to increases in their risk aversion), or both. Different default risk models have been proposed to characterize default events and explain the observed spread changes. Empirically, these possibilities are tested in studies of the informational content in credit spreads. One important

2. Tax effects may arise because, unlike government bonds, corporate bonds can be subject to state and local taxes on interest payments.

finding is that credit spreads do contain reliable information about time varying expected credit premiums.

DEFAULT RISK MODELS

Structural models of default are pioneered by Black and Scholes (1973) and Merton (1974). In this framework, firm value is modeled as a random process, and default is triggered when the firm value falls below some threshold.

While the structural approach has been faced with difficulty in explaining corporate bond prices and credit spreads, it produces default probabilities that are quite consistent with historical levels (Leland, 2004).

Reduced-form models like Duffie and Singleton (1999) treat default as an unpredictable event. The time to default is typically assumed to follow a random process, and different specifications of the process, as well as the recovery mechanism, lead to various models in this family.

These models have been widely used in the valuation of credit derivatives such as default swaps.

Fama (1986) is among the first to study private-issuer money market securities such as commercial paper and bankers' acceptances. Examining private-issuer securities relative to Treasury bills, he documented a positive relation between their forward rate spreads and subsequent return differences—wider spreads indicate higher average returns on private-issuer securities in excess of Treasury bills and vice versa.

Over the period of almost 150 years studied by Giesecke et al. (2011), the distribution of defaults is far from uniform and there have been many severe credit cycles characterized by high default rates.[3] However, credit spreads seem to have little explanatory power for default rates, also suggesting that variation in credit spreads is in large part driven by variation in expected credit premiums.

Similarly, Plecha and Rodriguez (2011) showed that credit spreads between intermediate corporate and government bonds (or low- and high-grade bonds) are informative about the difference in their expected returns. Nozawa (2014) proposed a decomposition framework that relates the cross-sectional variation of corporate bond spreads to changes in expected credit loss and expected returns. According to his study on 40 years of US corporate bond data, for investment grade bonds, over 90% of the variation in credit spreads can be attributed to variation in expected credit premiums.

As with term premiums, data (both live and simulated) shows that information in credit spreads can be used to build strategies that dynamically target credit premiums. When credit spreads are wide and expected premiums are large, a strategy that employs variable credit may increase its allocation to lower credit quality securities; when credit spreads are narrow and expected premiums are small, a greater focus may be applied to high-quality securities. By using market-based information in current credit spreads, the variable credit approach can be a reliable way to increase the return potential on the fixed income allocation for a given level of expected credit risk. Plecha and Rodriguez (2011) provided a summary of these results and showed how credit spreads between intermediate corporate and government bonds (or low- and high-grade bonds) are informative about the difference in their expected returns.

III. ADVANCES IN THE MARKETS

The introduction and expansion of the electronic fixed income trade database—known as TRACE—has been an important advance in the fixed income markets. Unlike stocks, most bonds are traded in secondary over-the-counter (OTC) markets, and prior to TRACE their transaction prices were not widely published. The increased availability of trade data has facilitated research into trading costs in bond markets, and more importantly, the impact of price transparency on bond trading.

Edwards, Harris, and Piwowar (2007) documented a size pattern in transaction costs in the corporate bond market. Estimated from TRACE data over the 2003–2005 period, effective half-spreads averaged 75 basis points for trade sizes smaller than $10,000 and decreased significantly with trade size—to 3 basis points for trade size of $5 million. Harris and Piwowar (2006) found a similar trade size and costs

3. Default rate is defined as the fraction of the total par value of the corporate bond market that enters into financial distress during each year.

relation in the municipal bond market. The results suggest that large institutional-sized trades allow for considerable price advantages over retail-sized trades.

TRACE is the Transaction Reporting and Compliance Engine. First rolled out in July 2002, the system captures and disseminates consolidated information on secondary market transactions in publicly traded, TRACE-eligible corporate bonds, closing the information gap between customers and dealers.

TRACE is a Financial Industry Regulatory Authority, Inc. (FINRA) developed vehicle. All broker-dealers who are FINRA member firms have an obligation to report transactions in corporate bonds to TRACE under a Securities & Exchange Commission (SEC) approved set of rules.

Another important finding is that trading costs for bonds have dropped considerably due to the price transparency attributable to TRACE. Bessembinder, Maxwell, and Venkataraman (2006) compared bond trades by insurance companies during the six months before and six months after the rollout of TRACE in July 2002.[4] Using a regression-based methodology, they found a 40% to 60% reduction in estimated trade costs post-TRACE. Edwards, Harris, and Piwowar (2007) focused on the period from 2003 to 2005, when TRACE phased in public dissemination of transaction prices, thereby increasing aggregate bond trades' transparency.[5] They reported that transaction costs for bond trades that had higher transparency were lower across all trade size groups after controlling for other relevant factors. Goldstein, Hotchkiss, and Sirri (2007) conducted a control study on a sample of BBB bonds whose transaction prices were made more transparent in April 2003 and a non-disseminated control sample matched on bond characteristics. They concluded that over the six-month window surrounding the start of dissemination on TRACE, increased transparency had not affected trading volume and led to a decline in effective spreads.

With TRACE, investors in the US corporate bond market gained some pre-trade and post-trade transparency that equity investors had enjoyed for decades. As a result, costs not only can be better measured, they are also considerably lower—effectively increasing investors' net-of-cost returns. In addition, price transparency also has a positive impact on the implementation of flexible fixed income strategies—such as the strategies Dimensional manages. Because TRACE provides market information that can be used to identify opportunities of potential higher premiums, evaluate credit ratings,[6] and facilitate a flexible and patient trading approach, it can increase the ability of a flexible trading strategy to reduce trading costs.

IV. INFLATION

Inflation drives a wedge between nominal and real returns, the latter of which measures the changes in purchasing power. Because the preservation of purchasing power can be one of the primary goals for investors, it is important to understand the behavior of inflation and how to cope with inflation uncertainty.

Real return is the rate of return on an investment after adjusting for inflation.

Purchasing power refers to the quantity of goods or services that can be purchased with a unit of currency. All else equal, inflation decreases one's purchasing power because the price level of goods and services has increased.

Inflation tends to be persistent, but can also experience sudden shifts. Over the 1952–2004 sample period, Ang, Bekaert, and Wei (2008) identified a number of switches among different inflation regimes characterized by the level (low or high) and variability (stable or volatile) of inflation. Despite the numerous forecasting methods proposed in the literature, it remains a challenge to accurately predict future inflation, and predictability is often limited to the short term. Using data from 1953 to 1971, Fama (1975) found that both past inflation rates and short-term nominal rates have some predictive power for inflation rates one to six months ahead. Ang, Bekaert, and Wei (2007) conducted a horse race of a comprehensive set of methods for forecasting annual inflation rates. While forecasting models—such as time series, macroeconomic, and term structure models—have not outperformed estimates based on survey data, neither

4. They relied on the National Association of Insurance Commissioners (NAIC) database of insurance company bond trades, which is the only publicly available database that contains corporate bond transaction information prior to TRACE.

5. By the end of their sample period, 99% of all trades in corporate and US agency debt were disseminated within 15 minutes, representing 95% of the dollar value traded.

6. Current yields provide an up-to-date source of credit rating information.

type of forecast has been particularly accurate. Evans and Wachtel (1993) documented that survey measures of inflation have tended to lag behind changes in actual and expected inflation. For example, the forecasts have systematically underestimated realized inflation through the 1970s as inflation accelerated and overestimated realized inflation during the disinflation of the 1980s.

Unexpected changes in inflation have the potential to erode the real value of invested wealth—this is a risk that nominal assets expose investors to. Nominal yields incorporate expectations of future inflation, so if actual inflation turns out to be higher, or if inflation expectation increases, nominal bond holders can experience a loss. Because inflation affects the entire economy and inflation risk cannot be diversified away, it is plausible that investors also require a positive premium for bearing the risk. This implies the difference between nominal and real yields—known as the breakeven inflation rate—can reflect both expected inflation and inflation risk premiums.[7] Because these components are not directly observable, researchers have used different methods to estimate them. Many of these studies have a relatively short history due to data limitations.[8]

Based on the real term structure derived from the prices of UK nominal and index-linked bonds, Evans (1998) documented evidence of a time-varying inflation risk premium over the 1983–1995 period. Ang, Bekaert, and Wei (2008) developed a model to identify the components embedded in the nominal yield curve. From 1952 to 2004, the estimated inflation risk premiums in the US have been varying through time and had an upward-sloping term structure—bonds with longer maturities tend to carry higher inflation risk premiums. Through a study of TIPS data, Grishchenko and Huang (2008) estimated the 10-year inflation risk premium to be around 15 basis points for the 2004–2008 period and also time varying. Pflueger and Viceira (2011) examined both TIPS and UK index-linked bonds, and in addition to confirming the time variation in inflation risk premiums, they found that inflation breakeven spreads explain only 3%–8% of the variation in excess returns on nominal bonds over real bonds. Taken together, the evidence suggests that nominal bond yields incorporate a positive inflation risk premium that exhibits variation across maturities and through time (although not in a reliably predictable way).

It is natural for investors to be concerned with the negative effect of inflation on their purchasing power. Those who are highly sensitive to inflation may find hedging against inflation risk a reasonable choice. To achieve this goal, one should place more emphasis on assets that move closely with inflation and may deliver stable real returns. In a study covering 45 countries, Bekaert and Wang (2010) showed that securities like nominal government bonds, equities, and real estate are poor inflation hedges. Securities such as TIPS, inflation swaps, or short-term bonds are a more effective tool for investors who want to hedge inflation (Lee and Lee, 2009; Goodrum, 2014). In doing so, however, investors are likely to give up some expected returns in exchange for the inflation insurance—the cost of hedging against inflation risk is effectively the inflation risk premiums demanded by nominal bond holders.

V. FOREIGN EXCHANGE

Yield curves vary across countries. It is not just that they move differently; the yield curves themselves have different shapes and nominal levels. This provides important diversification benefits and expands the investment opportunity set. Apart from what has been discussed so far, an additional consideration when it comes to investing in bonds issued in foreign currencies is the role of foreign exchange.

> **Covered interest rate parity (CIRP)** states that the difference in interest rates between two countries is equal to the differential between the forward exchange rate and the spot exchange rate, or the **forward currency premium.**

The covered interest rate parity (CIRP) condition is an arbitrage condition and holds generally in an open economy. It means that investors can enter into a currency forward contract to hedge against exposure to currency volatility. Because forward currency premiums are related to short-term interest rate differentials between the home market and foreign markets, hedging the currency exposure

7. Studies also document a potential liquidity premium (e.g., Pflueger and Viceira, 2011).
8. For example, it is generally believed that prior to 2004 the illiquidity in the TIPS market may bias the estimates of the inflation risk premium.

effectively replaces the foreign interest rate level with the domestic interest rate level at the maturity of the forward currency contract. Buying foreign bonds on a hedged basis provides investors with exposure to the shape of the foreign yield curve and therefore the opportunity to earn foreign term premiums—without having to bear the increased volatility associated with currencies.

As mentioned, Kolerich and Rodriguez (2011) provided a comprehensive exposition of the diversification benefits of a global, currency hedged, bond strategy over a bond strategy that focuses on just one yield curve and how such a strategy can potentially increase expected returns by pursuing term premiums globally.

Uncovered interest rate parity (UIRP) states that the difference in interest rates between two countries is equal to the expected change in exchange rates between the countries' currencies.

Under this condition, the high-interest rate currency is expected to depreciate against the low-interest rate currency. Historical data on interest rate differences and currency movements strongly rejects UIRP over short horizons.

What about buying foreign bonds on an unhedged basis? The uncovered interest rate parity (UIRP) condition hypothesizes that the expected change in exchange rates should just offset the interest rate differential, so investors should be indifferent among interest rate levels across countries. However, Hansen and Hodrick (1980) and Fama (1984c) found no evidence that the UIRP holds empirically. Their studies showed that when higher interest rates are observed in foreign markets, there is no tendency for the domestic currency to appreciate—in fact, it is even more likely to depreciate—relative to foreign currencies. In addition to the failure of UIRP, Meese and Rogoff (1983) investigated a variety of exchange rate models and found none of them had better forecasting power than simply using the past rate. More recent models reviewed in Rogoff and Stvrakeva (2008) also failed to produce robust forecasts of exchange rates at short horizons.

DECOMPOSING GLOBAL BOND RETURNS
(along the term dimension only)

Domestic bond return = domestic short-term interest rate + domestic term premium

Hedged foreign bond return = domestic short-term interest rate + foreign term premium

Unhedged foreign bond return = foreign short-term interest rate + foreign term premium + currency return

In the absence of the ability to predict exchange rate movements consistently, O'Reilly and Fogdall (2012) provided a framework that highlights the key ingredients for making informed currency hedging decisions. They showed that, because short-term expected currency returns are not reliably different from zero, forward currency premiums help inform the expected return difference between hedged and unhedged positions in foreign assets. Besides return, volatility is likely to be another important consideration. While hedging currencies manages volatility due to currency fluctuations, its impact on total volatility largely depends on how volatile a foreign asset's returns are relative to currency. In general, because equity returns tend to be more volatile than currency returns, unhedged and hedged volatilities for equity portfolios are similar; whereas because investment grade bond returns tend to be much less volatile than currency returns, the volatility of an unhedged global investment grade bond strategy is largely driven by exchange rate volatility, and therefore can be much larger than the volatility of a currency hedged bond strategy. Based on this evidence, hedging foreign exchange exposure may be appropriate for investors who prefer lower volatility from their global fixed income investments. For those who are willing to accept more volatility in pursuit of higher expected returns, they can opportunistically leave the currency exposure of certain bonds unhedged when the forward currency premium is negative. This approach can increase expected returns, does not rely on forecasting exchange rates, but increases expected volatility.

VI. CONCLUSIONS

Investors use fixed income securities to achieve a variety of investment goals. These goals can generally be divided into four broad categories: total return, customization of the overall portfolio risk profile, liability management, and capital preservation in real or nominal terms. More than four decades of research (by both academics and practitioners) suggest that the duration, credit quality, and diversification of a fixed income strategy are among the main considerations an investor must take into account when choosing a strategy designed to achieve those goals. Research also indicates that investors can use information in global yield curves and credit spreads to increase expected returns and/or manage risk within duration and credit quality ranges that are consistent with those goals. For example, an investor who desires capital preservation can use information in current prices in an attempt to dynamically capture the time-varying term and credit premiums among global, short-term, high quality bonds. In this way the investor may strike a better tradeoff between expected return, expected volatility, and the capital preservation goal.

One very real and practical implication of the research into fixed income markets is that the duration ranges, yield curves, and credit quality ranges a strategy invests in can all be customized to meet individual objectives and constraints and help investors better achieve their goals.

APPENDIX

This section provides a brief review of the basic concepts of fixed income instruments, including yield, holding period return, forward rate, and duration.

Yield

The yield to maturity of a bond is often referred to as simply its yield. It is the discount rate at which the present value of the sum of all future cash flows from a bond (coupons and principal) equals the current bond price. To calculate the yield, we solve for Y in the following bond price equation:

$$Bond\ price = \sum_{t=1}^{n} \frac{Coupon}{(1+Y)^t} + \frac{Par\ value}{(1+Y)^n}$$

Effectively, the yield is the internal rate of return on an investment in the bond if the bond is held to maturity and all coupons are reinvested in an interest rate equal to the bond's yield. It is worth noting, however, that if the reinvestment rate changes, or if a bond is not held to maturity, the realized holding period return (defined below) may be very different from the initial yield calculated upon purchasing the bond.

Similarly, we can find the continuously compounded yield *y* using the following equation:

$$Bond\ price = \sum_{t=1}^{n} Coupon \times e^{-yt} + Par\ value \times e^{-yn}$$

The continuously compounded version of yields and returns is used in the numerical examples in the main body because it often makes the calculation easier.[9]

Holding Period Return

The following formulas define the holding period return, realized by buying a bond at time t and selling it at time t + 1:

$$R = \frac{\$back}{\$paid} - 1$$
$$= \frac{Coupon + Reinvestment\ interest + Bond\ price_{t+1}}{Bond\ price_t} - 1$$

where the bond prices include accrued interest.

Taking the logarithm of the above, we obtain the formula for the continuously compounded return:

$$\mathrm{r} = \ln(Coupon + Reinvestment\ interest + Bond\ price_{t+1}) - \ln(Bond\ price_t)$$

As mentioned, the holding period return on a bond is a different concept from the bond's yield calculated at time t. It depends on the shape of the yield curve as well as its changes over time. While the realized holding period return is only observable at time t + 1, it turns out that the current yield curve contains useful information about its *expected* portion, in particular the differences in *expected* returns between different duration (defined below) bonds. This is discussed in detail in the main body of the paper.

9. For example, if the continuously compounded return is r_1 for the first period and r_2 for the second period, then the two-period return is obtained by simply adding them up ($r_1 + r_2$), which is more convenient than multiplying them as in the case of periodic compounding.

Forward Rate

We will use zero-coupon bonds—bonds that pay the face value at maturity with no intermediate payments—to define forward rates. The concept is the same when extended to more general cases, although the calculation may be more involved.[10]

Suppose Y_{n-1} and Y_n are the yields of zero-coupon bonds with *n-1* and *n* years to maturity, respectively. The forward rate for year *n* is defined by:

$$(1+Y_n)^n = (1+Y_{n-1})^{n-1}(1+F_n) \leftrightarrow F_n = \frac{(1+Y_n)^n}{(1+Y_{n-1})^{n-1}} - 1$$

Under continuous compounding, the definition becomes:

$$ny_n = (n-1)y_{n-1} + \mathrm{f}_n \leftrightarrow \mathrm{f}_n = ny_n - (n-1)y_{n-1}$$

In this formulation, the forward rate is defined as a "future" interest rate in year *n* that equates the return on an *n*-year zero-coupon bond, to that of an (*n* -1)-year zero-coupon bond rolled over into a one-year bond in year *n*. Therefore, forward rates are high when the yield curve is steeply upward sloping, i.e., yields are higher on longer maturity bonds than that on shorter maturity bonds.

Duration

The duration of a bond measures the sensitivity of the value of the bond to a one-time parallel shift in the entire yield curve. The longer the duration, the more sensitive a security's price is to interest rate changes.

Mathematically, suppose an *n* -year bond promises a stream of future cash flows, CF_t at time $t = 1, \ldots, n$, then the so-called Macaulay duration is defined by:[11]

$$D = \sum_{t=1}^{n} w_t \times t, \text{ where } w_t = \frac{CF_t/(1+Y)^t}{Bond\ price}$$

That is, the bond duration is a weighted average of the "maturity" of each of the bond's payments. The weight associated with the payment at time *t* is the ratio of the present value of that payment (discounted by the yield), to the total present value of the bond, i.e., the current bond price.

The duration of a zero-coupon bond is simply its time to maturity. This is because there are no intermediate payments, and the only payment is made at maturity. More generally, the duration of a bond is between zero and the maturity of the bond. The duration of a portfolio is the weighted average duration of all the bonds in the portfolio weighted by their dollar values.

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10. To calculate forward rates from data on coupon-bearing bonds, one can first derive the zero-coupon yield curve. The technique is based on the fact that, by treating each payment (coupon or principal) from a coupon-bearing bond as a zero-coupon bond that matures at the payment time, any coupon-bearing bond is effectively a "portfolio" of zero-coupon bonds.

11. The Macaulay duration can be similarly defined using continuously compounded yields: $d = \sum_{t=1}^{n} \frac{CF_t e^{-yt}}{Bond\ price} \times t$. There are also other duration measures that quantify different aspects of price sensitivity.

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